Grasshopper Tours

material from Václav Kotěšovec

The **Grasshopper** moves along queen lines, but must jump one man (the "hurdle") and land on the square immediately beyond (so a G on an empty board cannot move). The hurdle may be of either colour.

The idea of a grasshopper tour seems first to have been broached by T. R. Dawson in the January 1927 issue of the *Chess Amateur*. He set the task of placing the fewest possible number of hurdles on the board so that a grasshopper could tour the remaining squares, visiting each square once and once only and using each of the hurdles at least once. He called it a "formidable exercise" and gave no solution in the *Chess Amateur*, but he came back to it in *Fairy Chess Review* in August 1950 and gave a 39-move open tour over 24 hurdles. This has proved to be a long way short of the maximum possible. With the aid of a computer, Václav Kotěšovec has found a 52-move open tour over 11 hurdles,

9	48	*	10	34	16	47	19
3	*	4	28	2	*	29	*
45	49	36	*	37	17	33	20
12	*	13	11	5	15	*	14
35	50	46	27	39	21	30	18
1	**52**	7	44	*	8	32	43
23	*	24	22	38	26	*	25
6	51	**0**	41	*	42	31	40

and a 52-move closed tour over 12 hurdles :

49	43	51	*	50	38	22	14
7	*	8	47	4	6	*	5
52	44	28	*	29	27	23	37
20	42	3	48	9	*	21	*
16	*	17	13	24	39	30	15
2	45	19	33	*	26	32	36
*	41	*	40	10	12	*	11
1	34	18	46	*	35	31	25

Both are proven maxima. I number open tours from 0, closed tours from 1, so that the highest number always gives the number of moves in the tour.

The idea of a grasshopper tour over a single movable hurdle appears first to have been broached by S. H. Hall in *Fairy Chess Review* in February 1938. He used a knight, the knight and grasshopper moving alternately, but we'll come back to this later because his solution wasn't quite optimal and there has been progress since. It prompted Dawson to try the task using a rook (April 1938), which gave a very elegant solution :

50	48	53	47	55	36	49	34
1	26	8	28	11	30	10	32
52	46	51	45	54	35	56	37
7	27	6	25	9	29	12	31
63	44	61	41	57	38	59	39
5	24	3	22	13	19	14	20
64	42	62	43	60	40	58	33
2	17	4	23	15	21	16	18

The grasshopper starts at a7, the rook at a2, and we play Ga1, Rb2, Gc3, Rc2, Gc1, Rb2, Ga3, and so on. At the end, with the grasshopper on a4, we play Ra3, Ga2, Ra6, Ga7, Ra2, and round we go again. I have renumbered to highlight the pattern (Dawson started from Ga1/Rb2). We start by visiting the dark squares on the odd files, starting at a7 and ending at g1. From there, we go to b1 and visit the light squares on the even files, and so on round. Numbers in diametrically opposite squares always differ by 32.

In *Chessics* 5 (July 1978), C. M. B. Tylor looked at the task with a king, and found a solution with a similar four-fold symmetry :

62	52	61	50	42	48	41	46
57	55	59	56	37	35	39	36
64	51	63	49	44	47	43	45
58	53	60	54	38	33	40	34
2	8	**1**	6	22	28	21	26
13	11	15	12	17	31	19	32
4	7	3	5	24	27	23	25
14	9	16	10	18	29	20	30

This time we work round the board quarter by quarter, starting with Gc4 and Kb4, visiting each square in the lower left quarter finishing at c1, then moving to e3 and visiting each square in the lower right quarter, and so on.

Now to the knight. Hall, working by hand, found a 60-move open tour :

55	–	49	16	54	19	48	15
1	25	44	59	24	36	43	58
50	13	53	30	47	14	29	22
8	41	18	56	42	57	17	35
51	26	45	12	27	20	**60**	11
2	40	6	31	23	37	5	34
–	–	52	9	46	10	28	21
7	32	3	39	4	33	**0**	38

Nobody else, not even Dawson, did better than 57. This remained the target until T. H. Willcocks produced a 63-move open tour (*Chessics* 23, Autumn 1985) :

13	44	9	53	10	52	1	56
34	47	5	15	58	48	4	23
8	54	12	43	**0**	55	11	51
41	18	33	62	19	22	61	30
14	45	6	3	46	49	2	57
35	27	39	16	59	28	38	24
7	20	32	42	31	21	**63**	50
40	17	36	26	37	25	60	29

Additionally, the tour from **1** round to **62** is cyclic.

Willcocks, like Hall, appears to have worked by hand. Václav has now found a 64-move closed tour by computer :

31	34	26	41	32	35	25	40
44	55	62	20	45	9	2	19
27	23	30	49	24	39	48	36
61	17	43	54	33	18	42	8
28	50	14	21	46	51	13	37
60	56	63	4	57	10	3	7
15	22	29	53	16	38	47	52
1	5	59	12	**64**	6	58	11

Although closed, this is not cyclic (we end with the knight on b1, and cannot go round again). Václav tells me that the existence of a cyclic 64-move tour, although not yet proved impossible, appears unlikely.

All this, and much more, is in a book "Dual-free Leaper and Hopper Tours" recently produced by Václav in Praha (Prague). It is in Czech, but notes in English highlight what is going on, and the tables and diagrams are self-explanatory.